Foundations of Discrete Mathematics
COT 2104
Summer 2008

Homework 3

NOTE: This homework will be collected in three stages. The corresponding deadline appears before each set of questions. The final grade of this homework will covered the results of the three stages.

Deadline: 07/14/08 (at the beginning of class)

1. Determine which characteristics of an algorithm the following procedures have and which they lack.

    a. procedure divide(n: positive integer)
    b. while n ≥ 0
    c. begin
        i. m = 1/n
        ii. n = n – 1

    d. procedure chose(a, b: integers)
    e. x = either a or b

2. Describe an algorithm which given n real numbers, a_1, a_2… a_n outputs the number of a_i, which lie in the range 85-90, inclusive.

3. Solve the polynomial f(x) and each value of x using Horner's algorithm.

    a. $f(n) = 3x^2 + 1$; $x = 5$
    b. $f(n) = 2x^3 – 4x^2 – 7$; $x = 3$

4. To establish a big-Oh relationship find the witnesses c and k such that $|f(x)| \leq c|g(x)|$ when ever $x > k$. Determine whether each of these functions is O(x).

    a. $f(x) = 5x + 3$
    b. $f(x) = \lceil x/2 \rceil$

5. Determine whether each of these functions is $O(x^2)$.

    a. $f(x) = 17x + 11$
    b. $f(x) = x^2 + 1000$
    c. $f(x) = \lfloor x \rfloor \lceil x \rceil$

6. Use the definition of big-Oh given in class to show that f = O(g) in each of the following cases of functions f, g: N → R
    a. $f(n) = 2n^2 + 3n + 1$, $g(n) = n^2 + 1$

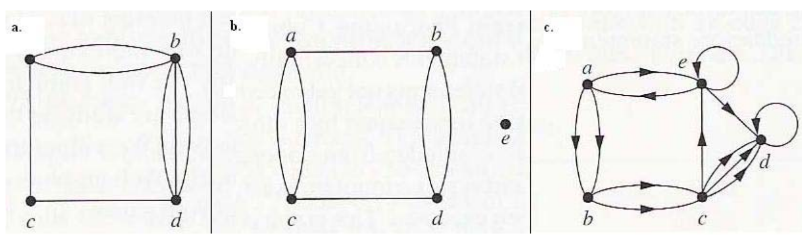
Deadline: before or by 07/21/08 at the beginning of the class

7. Draw graph models stating the type of graph used, to represent airline routines where every day there are four flights from Boston to Newark, two flights from Newark to Boston, three flights from Newark to Miami, two flights from Miami to Newark, one flight from Newark to Detroit, two flights from Detroit to Washington, two flights Washington to Newark, and one flight from Washington to Miami, with

    a. an edge between vertices representing cities for each flight that operates between them (in either direction).
    b. an edge from a vertex representing a city where a flight start to the vertex representing the city where it ends.

8. Determine whether the graph shown is a simple graph, a multigraph (and not a simple graph), a pseudograph (and not a multigraph), a directed graph, or a directed multigraph (and not a directed graph).

    a.

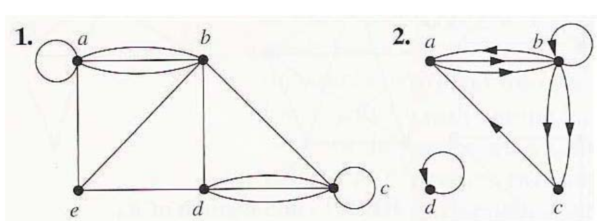


9. The intersection graph of a collection of sets A_1, a_2, …, A_n is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.

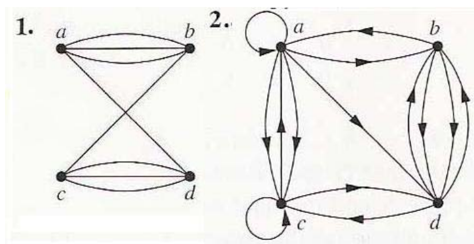
    a. $A_1 = \{x| x < 0\}$, $A_2 = \{x| -1 < x < 0\}$, $A_3 = \{x| 0 < x < 1\}$, $A_4=\{x | -1 < x < 1\}$, $A_5= \{x | x > -1\}$, $A_6 = \mathbf{R}$

10. Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated vertices. If the graph is directed multigraph find the in-degree and out-degree of each vertex.

    a.

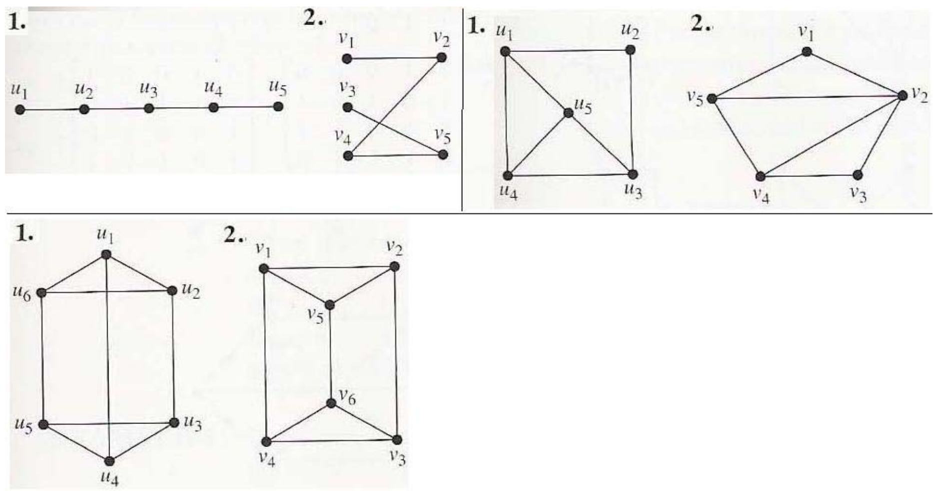


11. Represent the adjacency matrix of the following graphs



a.

12. Determine whether the given pair of graph is isomorphic

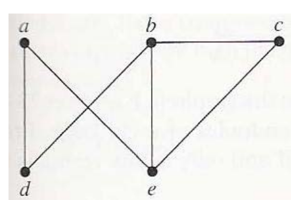


Deadline: before or by 07/28/08 at the beginning of the Final Test

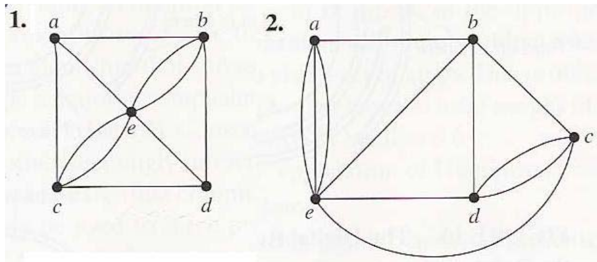
13. Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are path?

a) a, e, b, c, b

b) e, b, a, d, b, e



14. Determine whether the given graph has an Euler circuit. Construct such a circuit when one exits. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

a. 

15. Find the length of a shortest path between a and z in the given weighted graph.

a.